Problem Set 9: Large Scale Structure in Cosmology

Astrophysics and Cosmology through Problems

Due 27 Nov. 2008

The Fourier transform of the cosmological overdensity field $\delta(\mathbf{x})$, at any given time, is defined as:

$$\delta_{\mathbf{k}} \equiv \int d^3\mathbf{x} \exp(-i\mathbf{k}\cdot\mathbf{x})\delta(\mathbf{x}). \tag{1}$$

If $\delta(\mathbf{x})$ is a statistically homogenous and isotropic gaussian random field, all its statistics can be described in terms of its auto-power spectrum $P(k)$, where:

$$\langle \delta_{\mathbf{k}}\delta^*_{\mathbf{k}'}\rangle = (2\pi)^3 P(k)\delta^3(\mathbf{k}-\mathbf{k}') \tag{2}$$

1. Show that the two-point correlation function is the Fourier transform of the power spectrum:

$$\xi(r) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\exp(i\mathbf{k}\cdot\mathbf{r})P(k) \tag{3}$$

2. Show that the variance of the mean overdensity within a sphere of radius R (and mass $M = \frac{4\pi\bar{\rho}R^3}{3}$) is:

$$\sigma^2_R = \sigma^2(M) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}|W(kR)|^2 P(k). \tag{4}$$

   What is $W(kR)$? Verify that for a random Poisson distribution of galaxies, $P(k) = n_g^{-1} =$ const., where n_g is mean number density of galaxies.

3. From inflationary initial conditions, the power spectrum can be well-fit by the BBKS form:

$$P(k = q\Omega_m h^2 \mathrm{Mpc}^{-1}) = Aq\left[\frac{\ln[1+2.34q]}{2.34q}\right]^2 \times \left[1 + 3.89q + (16.2q)^2 + (5.47q)^3 + (6.71q)^4\right]^{-1/2}. \tag{5}$$

   WMAP5 data show $\Omega_m = 0.27$, $h = 0.7$ and $\sigma_{8h^{-1}\mathrm{Mpc}} = 0.8$. Find the normalization A numerically and plot $\sigma(M)$ and σ_R as functions of mass ($10^{10}M_\odot < M < 10^{16}M_\odot$) and radius (1 Mpc $< R < 10^4$Mpc) respectively.

4. Argue that the probability of the a region of mass M, collapsing before today is:

$$\mathcal{P}(M) = \int_{\delta_c}^{\infty} \frac{d\delta}{[2\pi\sigma^2(M)]^{1/2}} \exp\left[\frac{-\delta^2}{2\sigma^2(M)}\right], \tag{6}$$

   where $\delta_c \simeq 1.68$ is the linear collapse threshold. Press and Schechter hypothesized that the fraction of the mass of the Universe collapsing into haloes more massive than M is:

$$f(>M) = 2\mathcal{P}(\mathcal{M}), \tag{7}$$

   where the factor 2 ensures that $f(>0) = 1$. Using this hypothesis, find and plot the mass function (number density per unit mass) of haloes in the mass range $10^{10} - 10^{16}\ M_\odot$.